

K23P 0501

Reg. No. :

Name :

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT2C09 : Foundations of Complex Analysis

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Evaluate the integral $\int_{\gamma} \frac{dz}{z^2 + 1}$, where $\gamma(t) = 2e^{it}$, $0 \leq t \leq 2\pi$.
2. Define index of a closed rectifiable curve with respect to a point. Illustrate with example.
3. Determine the nature of the singularity of the function $f(z) = \frac{\log(z+1)}{z^2}$.
4. Find the residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$.
5. Define pole and essential singularity, giving one example of each.
6. State Arzela-Ascoli theorem in space of continuous functions.

PART – B

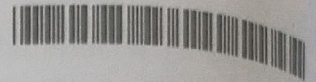
Answer **any 4** questions without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) If G is a region and $f : G \rightarrow \mathbb{C}$ is an analytic function such that there is a point a in G with $|f(a)| \geq |f(z)|$ for all z in G . Prove that f is constant.
b) If G is simply connected and $f : G \rightarrow \mathbb{C}$ is analytic in G . Prove that f has primitive in G .

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8. a) Let γ be a closed rectifiable curve in \mathbb{C} . Prove that $n(\gamma, a)$ is constant in each component of $\mathbb{C} - \gamma$.
- b) Evaluate the integral $\int_{\gamma} \frac{(e^z - e^{-z})dz}{z^n}$, where n is a positive integer and $\gamma(t) = e^{it}$, $0 \leq t \leq 2\pi$.
9. State and prove Goursat's theorem.

Unit – II

10. a) Find the Laurent series expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in $\text{ann}(0, 1, 2)$.
- b) Let $z = a$ be an isolated singularity of f and let $f(z) = \sum_{n=-\infty}^{\infty} a_n(z-a)^n$ be its Laurent expansion in $\text{ann}(a, 0, R)$. Prove that $z = a$ is a pole of order m if and only if $a_m \neq 0$ and $a_n = 0$ for $n \leq -(m+1)$.
11. a) Evaluate the integral $\int_0^{\pi} \frac{d\theta}{a + \cos \theta}$, $a > 1$ by the method of residues.
- b) State and prove Argument principle.
12. Let $D = \{z : |z| < 1\}$ and suppose f is analytic on D with $|f(z)| \leq 1$ for z in D and $f(0) = 0$. Prove that $|f'(0)| \leq 1$ and $|f(z)| \leq |z|$ for all z in the disk D .

Unit – III

13. a) Prove that $C(G, \Omega)$ is a complete metric space.
- b) If $\mathcal{F} \subset C(G, \Omega)$ is equicontinuous at each point of G . Prove that \mathcal{F} is equicontinuous over each compact subset of G .
14. a) State and prove Hurwitz's theorem.
- b) Let $\text{Re} z_n > -1$, prove that the series $\sum_{n=1}^{\infty} \log(1+z_n)$ converges absolutely if and only if the series $\sum_{n=1}^{\infty} z_n$ converges absolutely.
15. State and prove Weierstrass Factorization theorem.